

# Modern Algebraic Geometry

July 23-26, 2018

Lecture Hall, Jiayibing Building, Jingchunyuan 82, BICMR, Peking University

## Talks

DAMIAN BROTBEK (University of Strasbourg)

### *Hyperbolicity properties of general complete intersections*

A complex manifold is said to be Brody hyperbolic if it doesn't contain any entire curve. Kobayashi conjectured in the 70s that if  $H$  is a general hypersurface of sufficiently large degree in  $\mathbb{P}^n$ , then  $H$  and its complement are both hyperbolic.

One way to control the hyperbolicity properties of a variety is to study the positivity properties of its cotangent bundle. Indeed a variety with ample cotangent bundle is hyperbolic. It is known that the cotangent bundle of hypersurfaces in  $\mathbb{P}^n$  is not ample, however, in a related direction, Debarre conjectured that that if  $X$  is a general complete intersection of sufficiently large multi degree in  $\mathbb{P}^n$  such that the codimension of  $X$  exceeds its dimension, then the cotangent bundle of  $X$  is ample.

The relation between those two conjectured is best understood using jet differentials, higher order generalizations of symmetric differential forms.

In this talk we will present a method for constructing jet differential equations on some special complete intersection varieties and explain how this can be used to prove the conjectures of Debarre and Kobayashi.

YOHAN BRUNEBARBE (University of Zurich)

### *Hyperbolicity of moduli spaces of Calabi-Yau varieties with level structure*

Moduli stacks of polarized complex varieties whose canonical bundle is trivial are known to enjoy many hyperbolicity properties. In this talk, I will explain how adding level structure to the picture yields much stronger results. The proofs are Hodge theoretic.

BAOHUA FU (AMSS & MCM, Chinese Academy of Sciences)

### *On Fano complete intersections in rational homogeneous varieties*

Complete intersections inside rational homogeneous varieties provide interesting examples of Fano manifolds. We first classify these Fano complete intersections which are locally rigid. It turns out that most of them are hyperplane sections. We then classify general hyperplane sections which are quasi-homogeneous. This is a joint work with Chenyu Bai and Laurent Manivel.

ADRIAN LANGER (University of Warsaw)

### *Drinfeld's half-space over finite fields*

The talk will be devoted to describing geometry of quotients of Drinfeld's half-space and their compactifications. I will try to relate this geometry to the action of Steenrod's algebra. I will also

study automorphism groups of Drinfeld’s half-space and show that they form non-trivial formal schemes.

RADU LAZA (Stony Brook University)

*Birational Geometry of the moduli space of  $K3$  surfaces*

I will discuss a program, joint with K. O’Grady, to investigate the birational geometry of locally symmetric varieties of  $K3$  type (similar considerations apply to the case of ball quotients).

The motivation for our study is the search for geometric compactifications for the moduli of polarized  $K3$  surfaces. Namely, as a consequence of Torelli theorem, the moduli of polarized  $K3$  surfaces (with canonical singularities) can be identified to a locally symmetric variety  $D/\Gamma$ . As such, there are natural “arithmetic” compactifications, e.g. the Baily-Borel (BB) compactification. Unfortunately, the BB compactification has obscure geometric meaning. Consequently, it is natural to compare it with more geometric compactifications, such as those given by GIT. I will explain that there is a natural continuous interpolation between BB and GIT compactifications, and that there is a rich geometric and arithmetic structure behind this picture. In particular, I will show that the Borchers-Gritsenko relations provide an explanation to some surprising geometric behavior.

The focus of the talk will be on the quartic  $K3$  case. Some new results on degree 6  $K3$ s (with F. Greer) will be briefly discussed.

BRIAN LEHMANN (Boston College)

*The exceptional set in Manin’s Conjecture*

Let  $X$  be a Fano variety over a number field and let  $L$  be an adelically metrized ample line bundle on  $X$ . Manin’s Conjecture predicts the growth rate of points of bounded  $L$ -height. After removing an “exceptional set”, the growth rate should be determined by geometric invariants comparing the positivity of  $L$  and  $-K_X$ . I will give a conjectural description of the exceptional set which includes the rational point contributions from all subvarieties and covers with larger geometric invariants. The main result is that this candidate set is contained in a thin set as predicted by Peyre. This is joint work with Akash Kumar Sengupta and Sho Tanimoto.

CHUNYI LI (University of Warwick)

*Bogomolov type inequality for Fano varieties with Picard number 1*

I will talk about some basic facts about slope stable sheaves and the Bogomolov inequality. New techniques from stability conditions will imply new stronger bounds on Chern characters of stable sheaves on some special varieties, including Fano varieties, quintic threefolds, etc. I will discuss the progress in this direction and some related open problems.

LINQUAN MA (University of Utah)

*Homological conjectures, perfectoid spaces, and singularities in mixed characteristic*

The homological conjectures have been a focus of research in commutative algebra since 1960s. They concern a number of interrelated conjectures concerning homological properties of commutative rings to their internal ring structures. These conjectures had largely been resolved for rings that contain a field, but several remained open in mixed characteristic—until recently Yves Andre proved Hochster’s direct summand conjecture and the existence of big Cohen-Macaulay

algebras, which lie in the heart of the homological conjectures. The main new ingredient in the solution is to systematically use the theory of perfectoid spaces, which leads to further developments in the study of mixed characteristic singularities. For example, using perfectoid algebras and big Cohen-Macaulay algebras, we can define the mixed characteristic analog of rational/ $F$ -rational and log terminal/ $F$ -regular singularities, and they have applications to questions on singularities over arithmetic families (based on recent joint work with Karl Schwede). In this talk, we will give a survey on these results and methods.

DRAGOS OPREA (University of California San Diego)

*Tautological integrals over the Hilbert scheme of points*

Recently, there has been progress in evaluating the generating series of Segre integrals of tautological vector bundles over the Hilbert scheme of points on surfaces. In rank 1, the Segre series were the subject of a conjecture of Lehn from 1999. Quite surprisingly, in arbitrary rank, the Segre series are also conjecturally related to the Verlinde-type generating series of Euler characteristics of tautological bundles over the Hilbert scheme. I will explain this circle of ideas, some of the conjectures and results. This is joint work with Alina Marian and Rahul Pandharipande.

PHUNG HO HAI (Institute of Mathematics Hanoi)

*Prudent group schemes over a complete discrete valuation ring*

An affine group scheme is completely determined by its coordinate ring, which is a (commutative Hopf algebra). Among flat affine group schemes over a Dedekind ring, those which have coordinate rings projective (over the base ring) have many good properties. We are looking for conditions for an affine group scheme over a Dedekind ring, or more specific, over a discrete valuation ring, to have projective coordinate ring. When the base ring is a complete discrete valuation ring, this property is equivalent to being prudent, a property that can be checked on each finite quotient of the base ring. We show various applications of this property.

This is a joint work with J. P. P. dos Santos (Paris).

STEFAN SCHREIEDER (University of Munich)

*Stably irrational hypersurfaces of small slopes*

We give an overview on what is known about the rationality problem for hypersurfaces in projective space. We then explain how to show that a very general complex projective hypersurface of dimension  $N$  and degree at least  $\log_2(N) + 2$  is not stably rational. The same statement holds over any uncountable field of characteristic  $p \gg N$ .

JUNLIANG SHEN (ETH Zurich)

*Special subvarieties in holomorphic symplectic varieties*

Every projective  $K3$  surface contains a rational curve due to a result of Bogomolov and Mumford. We will discuss a conjecture of Voisin concerning algebraically coisotropic subvarieties of holomorphic symplectic varieties, which can be viewed as a higher dimensional generalization of the Bogomolov-Mumford theorem. We show that Voisin's conjecture holds when the holomorphic symplectic variety arises as a moduli space of sheaves on a  $K3$  surface. Then we will discuss recent progress on the study of rational curves in holomorphic symplectic varieties, and the role

played by Gromov-Witten theory. Based on joint work with Georg Oberdieck, Qizheng Yin, and Xiaolei Zhao.

HIROMU TANAKA (University of Tokyo)

*On varieties of Fano type in positive characteristic*

The notion of varieties of Fano type is a generalisation of smooth Fano varieties, which is known as one of important classes in minimal model program. However, varieties of Fano type behave pathologically in characteristic  $p$ . In this talk, we first summarise fundamental properties of varieties of Fano type in characteristic zero. Then we will consider which properties should still hold true in characteristic  $p$ .

NIKOLAI VAVILOV (St. Petersburg State University)

*Structure of reductive groups over commutative rings*

SERGE YAGUNOV (St. Petersburg Department of Steklov Mathematical Institute/First Electrotechnical University)

*Homotopy invariance and its relatives: from classical to  $\mathbb{A}^1$ -topology*

The talk is dedicated to the memory of Andrei Suslin who passed away on July 10th, 2018.

The notion of homotopy invariance plays a key role in algebraic topology. It tells, roughly speaking, that (co-)homology functors cannot separate homotopy equivalent spaces. Passing to the case of algebraic geometry, one changes the unit interval with the affine line. Since there are too few rational curves on algebraic varieties, the resulted homotopy relation becomes very inconvenient. However, working with cohomology theories in the algebro-geometric case it is often possible (and useful) to consider a different property instead of the homotopy invariance. This property, called rigidity, was initially introduced by Andrei Suslin for algebraic  $K$ -functor and now it has found a lot of applications in many different cases.

In my talk I'll try to show the difference between the homotopy invariance and rigidity, give some applications of the rigidity property, and explain how it is connected to orientability of the algebraic cohomology theory.

TAKEHIKO YASUDA (Tohoku University)

*Quotient singularities by the cyclic group of order  $p$*

Consider the quotient variety associated to a linear action of the cyclic group of order  $p$  on an affine space in characteristic  $p$ . I will talk about a result determining when the quotient variety is terminal, canonical and log canonical. The main ingredients are motivic integration and the Artin-Schreier theory applied to the Laurent power series field  $k((t))$ .

XINWEN ZHU (California Institute of Technology)

*Bessel equations, Kloosterman sums, and the geometric Langlands correspondence*

I will first review the relationship between the classical Bessel equation

$$\left(x \frac{d}{dx}\right)^2 u - xu = 0,$$

and the classic Kloosterman sum

$$\text{Kl}(a) := \sum_{xy=a \pmod{p}} \exp\left(\frac{2\pi i}{p}(x+y)\right).$$

Such a relation can be regarded as an instance of the geometric Langlands correspondence for  $\text{GL}_2$ . I will then explain the recent generalizations of this story for arbitrary reductive groups, based on the works by Frenkel-Gross, Heinloth-Ngô-Yun, and myself. I'll particularly emphasize the geometry involved in the story, such as certain Hitchin systems with ramification. Time permitting, I will also discuss some joint work in progress with Daxin Xu, where we further explore the  $p$ -adic aspects of this story.

## Poster Sessions

ISWAR MANI ADHIKARI (Tribhuvan University, Nepal)

*Evacuation planning problems on transit-based networks*

DOMINIK BUREK (Jagiellonian University, Poland)

*Higher dimensional Calabi-Yau manifolds of Kummer type*

DAO VAN THINH (National University of Singapore)

*Hitchin fibration and Vinberg's invariant theory*

KHIM BAHADUR KHATTRI (Kathmandu University, Nepal)

*Extended quasi-two-phase mass flow model with simulations of flow-obstacle-interactions*

ROMAN LUBKOV (St. Petersburg State University, Russia)

*Overgroups of exterior powers of elementary groups*

GRZEGORZ MALARA (Pedagogical University of Cracow, Poland)

*Unexpected curves arising from special line arrangements*

SHENG MENG (National University of Singapore)

*On projective varieties admitting polarized endomorphisms*

ILIA NEKRASOV (St. Petersburg State University, Russia)

*Symmetric-exterior plethysms, vertex algebras and generalized Sato Grassmannians*

ANA NUSKHELADZE (Tbilisi State University, Georgia)

*Approximate solution of the Cauchy problem for abstract hyperbolic equations using Padé approximants for sine and cosine operator functions*

DMITRY OGANESYAN (Moscow State University, Russia)

*Deformations of Belyi pairs in Hurwitz spaces*

APRIL LYNNE D. SAY-AWEN (De La Salle University, Philippines)

*A family of aperiodic tilings with infinitely many orientations*

KONSTANTIN TSVETKOV (St. Petersburg State University, Russia)

*Geometry of unipotent elements in the Chevalley group of type  $E_6$*

GIORGI TUTBERIDZE (University of Georgia)

*On the boundedness of subsequences of Vilenkin-Fejer means on the martingale Hardy spaces*

ZURAB VASHAKIDZE (University of Georgia)

*Application of Legendre polynomials for numerical solution of Kirchhoff nonlinear equation*

EGOR VORONETSKY (St. Petersburg State University, Russia)

*Subgroups of the general linear group, normalized by the elementary unitary group*

MACIEK EMILIAN ZDANOWICZ (University of Warsaw, Poland)

*Serre-Tate theory for ordinary Calabi-Yau varieties*