

First Name	Last Name	Affiliation	Title	Abstract
David	Beltran	BCAM Basque Center for Applied Mathematics, Spain	Local smoothing estimates for wave equations via decoupling inequalities	<p>For fixed time, the sharp <math>L^p</math>-Sobolev space-estimates for the solution of the linear homogeneous wave equation in terms of the initial data are well known. The local smoothing phenomenon for the wave equation consists in a regularity gain for space-time estimates with respect to the fixed-time case after averaging over a compact interval of time. This was first observed by Sogge, who conjectured the sharp regularity gain depending upon the Lebesgue exponent.</p> <p>In the Euclidean setting, the best known results on the conjecture (for <math>n \geq 3</math> and up to the regularity endpoint) follow from the celebrated sharp <math>\ell^p</math>-decoupling (or Wolff-type) inequalities of Bourgain and Demeter for the Fourier extension operator for the cone. In this talk, we will discuss the recently obtained variable coefficient versions of the decoupling inequalities of Bourgain and Demeter, which imply new local smoothing estimates for wave equations on compact manifolds. This is joint work with Jonathan Hickman and Christopher D. Sogge.</p>
Nicolas	Burq	Universite Paris-Sud, France	TBA	TBA
Gong	Chen	Univeristy of Toronto, Canada	Strichartz estimates for linear wave equations with moving potentials	<p>We will discuss Strichartz estimates for linear wave equations with several moving potentials in <math>\mathbb{R}^3</math> (a.k.a. charge transfer Hamiltonians) which appear naturally in the study of nonlinear multi-soliton systems. To study local decay estimates which plays an imporant role to derive Strichartz estimates, we introduce novel reversed Strichartz estimates along slanted lines and energy comparison under Lorentz transformations. As applications, we will also discuss related scattering problems and a construction of multi-soliton in <math>\mathbb{R}^3</math> with strong interactions in a toy model.</p>

Renjie	Feng	BICMR, Peking University, China	Spectrum of SYK mode	The SYK model is a random matrix model arising from condensed matter theory in statistics physics and black hole theory in high energy physics. In this talk, we will first review some elementary results in random matrix theory, then we will introduce the SYK model. I will explain the spectral properties of the random matrix of SYK model, such as the global density where a phase transition is observed, the central limit theorem of the linear statistics and the concentration of measure theory. In particular, we will derive the large deviation principle when the number of interaction of fermions is 2. This is my joint work with G. Tian and D. Wei.
Chengchun	Hao	AMSS, CAS, China	On the motion of free boundary problem of ideal incompressible MHD	In this talk, based on a joint work with T. Luo, I will discuss the free boundary problem of ideal incompressible MHD flows in Sobolev norms by adopting a geometrical point of view and some quantities such as the second fundamental form and the velocity of the free boundary are estimated.
Daoyin	He	Fudan University, China	Long Time Behavior of Semilinear Tricomi Equations	On Page 10
Bobo	Hua	Fudan University, China	Some results on eigenvalues of Laplacians on graphs	Discrete Laplacians on graphs are analogs of Laplace–Beltrami operators on manifolds. In this talk, we discuss some problems related to spectra of discrete Laplacians. We will report some results on Cheeger estimates using isoperimetric inequalities on general graphs, and some universal inequalities on Dirichlet Laplacians on subgraphs of integer lattices.

Oana	Ivanovici	Universite de Nice, France	Dispersive estimates for the wave equation outside strictly convex obstacles	<p>We consider the linear wave equation outside a compact, strictly convex obstacle in <math>\mathbb{R}^d</math> with smooth boundary. For <math>d=3</math> we show that the linear flow satisfies the dispersive estimates as in the <math>\mathbb{R}^3</math>. In higher dimensions <math>d&gt;3</math> and if the obstacle is a ball, we show that there exists points where the dispersive estimates fail. This is joint work with Gilles Lebeau.</p>
Hao	Jia	University of Minnesota, USA	Universality of blow up for wave maps	<p>In the lectures, we aim to give an introduction to the regularity theory and certain dynamical issues of energy critical wave maps. For simplicity, we will focus on the case when the target is the two dimensional sphere. Wave map equations are extremely interesting mathematically, and the study of these equations has led to many deep ideas.</p> <p>Prominent ideas include:</p> <ol style="list-style-type: none"> <li>1. the use of well adapted spaces for the wave equations such as <math>X^{s,b}</math> (Klainerman, Machedon, Selberg) and null frame spaces (Tataru);</li> <li>2. the choice of favorable gauges such as microlocal gauge (Tao) to transform the nonlinearity into easier forms;</li> <li>3. monotonicity formulae especially those of Morawetz type for controlling large wave maps;</li> <li>4. energy induction or concentration compactness argument to prove the ground state conjecture (Sterbenz-Tataru, Tao, Krieger-Schlag).</li> </ol> <p>We will also emphasize a recent application of the channel of energy argument for outgoing waves in ruling out the so called “null concentration of energy” for wave maps. Presently the argument works only when the energy of the map is only slightly above the ground state, and gives universality of the blow up in that case. A particular consequence is the “quantization of energy” of blow up for wave maps-- each blow up costs exactly the energy of a traveling wave.</p> <p>We will first take a global view of the existing results and some open questions, then give a flavor of the perturbation theory and use the perturbation theory together with energy coercivity near traveling waves to give a proof of the universality of blow up, when the energy is close to the energy of the degree 1 harmonic map.</p>

Renjin	Jiang	Tianjin University, China	Recent developments on Riesz transform, harmonic functions and heat kernels	In this report, we shall report some recent developments regarding Riesz transform, harmonic functions and heat kernels on manifolds as well as general metric measure spaces. In a general framework of Dirichlet metric measure spaces, we shall provide characterizations of $L^p$ -boundedness of the Riesz transform, $L^p$ -gradient estimates for harmonic functions and heat kernels.
Long	Jin	Purdue University, USA	Control and stabilization on hyperbolic surfaces	In this talk, we discuss some recent results concerning the control and stabilization on a compact hyperbolic surface. In particular, we show that the Laplace eigenfunctions have uniform lower bounds on any nonempty open set; the linear Schrödinger equation is exactly controllable by any nonempty open set; and the energy of solutions to the linear damped wave equation with regular initial data decay exponentially for any smooth damping function. The new ingredient is the fractal uncertainty principle for porous sets by Bourgain - Dyatlov. This is partially based on joint work with Semyon Dyatlov, Kiril Datchev and Ruixiang Zhang.
Xudong	Lai	Harbin Institute of Technology, China	Multilinear estimates for the higher order Calderón commutators	In this talk, we introduce some recent results about the multilinear boundedness properties of the higher (n-th) order Calderón commutator for dimensions larger than two. All multilinear bounds on the products of $L^p$ spaces are established for Calderón commutators here. At the endpoints, things become quite different to the classical multilinear C-Z operators. Some new multilinear estimates on the products of Lorentz spaces are found.
Junfeng	Li	Beijing Normal University, China	The boundedness of Hilbert transform along variable curve	In this talk, I will present our recent results on the boundedness of Hilbert transform along variable curve. We are interested to set up the $L^p$ boundedness of the transform under certain general conditions on the curve. The main ingredients of the proofs are oscillatory estimate and TT* method.

Xiaochun	Li	University of Illinois at Urbana - Champaign, USA	Recent progress on Schrodinger equations	We will report some recent progress on the pointwise convergence of the solutions to Schrodinger equations. The two dimensional case was solved by Du, Guth, and me. Some higher dimensional improvements were obtained by Du, Guth, Zhang and me.
Baoping	Liu	BICMR, Peking University, China	Channel of energy estimate for linear wave equation with application	Channel of energy estimate was first considered in dimension 3 by Duyckaerts, Kenig and Merle, which played an important role in their works on the long time dynamics for wave equations. In this talk, I will discuss the general form of the channel of energy estimate in all odd dimensions for radial waves. I will also discuss its application in the exterior wave map problem.  This is based on joint works with Carlos Kenig, Andrew Lawrie and Wilhelm Schlag.
Angel	Martinez	Universidad Autonoma de Madrid, Spain	Fractional operators and SQG equation on the two dimensional sphere	Certain transport equations with (non local) velocity and critical diffusion had attracted attention during the last decades. Among them, the Surface Quasigeostrophic equation (SQG) has been studied extensively due to its relation with the 3D Euler equations. In the works of Kiselev, Nazarov and Volberg one one side, and Caffarelli and Vasseur on the other, different techniques were introduced to study the existence of strong global solutions for the SQG equation with critical dissipation. Later on, in a series of works due to Constantin, Vicol (and Tarfulea) a different strategy was introduced. We will introduce some basics about the equation. After that we will present the fractional Laplace-Beltrami operators on compact manifolds together with some of the basic properties they satisfy in relation to the problem of global existence of solutions for the SQG (i.e. Córdoba-Córdoba inequality, integral representation, etc.). Finally, we will sketch the proof of global existence of solutions in the case of the sphere.

Fabrice	Planchon	Universite de Nice, France	A parametrix construction for the wave equation in a convex domain and applications	We will review how to construct good high frequency approximations to waves gliding along a convex boundary, focusing on a model case that exemplifies all the key elements. We will then derive sharp pointwise bounds on the Green function and use them to get optimal dispersive estimates. If times permits, we will also use our parametrix to improve on known counterexamples, closing on the gap between positive and negative results, at least in some cases.
Ruipeng	Shen	Tianjin University, China	Scattering of solutions to the 3D defocusing energy sub-critical wave equation	On Page 11
Chris	Sogge	Johns Hopkins University, USA	Local smoothing of Fourier integrals	<p>My talks will serve as an introduction for David Beltran's talk on June 4.</p> <p>I shall present background on the local smoothing problem for Fourier integrals satisfying the ``cinematic curvature'' condition. I shall show how local smoothing estimates imply Bourgain's circular maximal theorem and go over early work on local smoothing. I shall also show how optimal local smoothing estimates for the Euclidean wave equation imply the Bochner-Riesz conjecture.</p> <p>I shall also go over how less optimistic results can hold for variable coefficient Fourier integral operators and indicate how the decoupling estimates of Bourgain and Demeter are useful.</p>

Chengbo	Wang	Zhejiang University	Concerning ill-posedness for semilinear wave equations	In this talk, we will investigate the problem of optimal regularity for derivative semilinear wave equations in $H^s$ , in general spatial dimension. Moreover, in the case of quadratic nonlinearity, we will establish a criteria on the structure for the problem to be locally well-posed for any $s > n/2$ for $n \leq 4$ . On the other hand, the problem was known be locally well-posed for higher spatial dimension. This is a joint work with Mengyun Liu.
Dongyi	Wei	BICMR, Peking University, China	Energy identity for approximate harmonic maps from surface to general targets	Let $u_n$ be a sequence of mappings from a closed Riemannian surface $M$ to a general Riemannian manifold $N \subset \mathbb{R}^k$ . If $u_n$ satisfies $\sup_n \left( \ \nabla u_n\ _{L^2(M)} + \ \tau(u_n)\ _{L^p(M)} \right) \leq \Lambda$ $\text{for some } p > 1$ , where $\tau(u_n)$ is the tension field of $u_n$ , then there hold the so called energy identity and neckless property during blowing up. This result is sharp by Parker's example, where the tension fields of the mappings from Riemannian surface are bounded in $L^1(M)$ but the energy identity fails.
Jim	Wright	University of Edinburgh, UK	$L^2$ Fourier Restriction Theorems	On Page 12

Emmett	Wyman	Johns Hopkins University, USA	Period integrals of eigenfunctions in nonpositively curved manifolds	We are interested in bounds on period integrals - integrals of Laplace eigenfunctions over curves in compact surfaces. These bounds tell us something about the degree to which eigenfunctions oscillate along a given curve. In manifolds with nonpositive curvature, the geometry of the curve affects the decay of the period integrals. In the flat torus, for example, period integrals over curves with nonvanishing curvature enjoy polynomial decay, while period integrals over straight lines do not in general decay at all. We focus on the situation where the manifold is a compact surface with nonpositive sectional curvature and determine that horocycles are the problematic curves in the hyperbolic setting. We also discuss a generalization to higher dimensions.
Lixin	Yan	Sun Yat-Sen University, China	Some results on Bochner-Riesz means for elliptic operators	In this talk we will investigate $L^p$ bounds for the Bochner-Riesz means and the maximal Bochner-Riesz operators for self-adjoint operators of elliptic type. In particular, we apply it to the Hermite oscillator $H = -\Delta +  x ^2$ in $\mathbb{R}^n$ and for other related operators, improving earlier results of Thangavelu and of Karadzhov.
Jianwei	Yang	BICMR, Peking University, China	Blow-up of a critical Sobolev norm for energy-subcritical and energy-supercritical wave equations	On Page 13

Steve	Zelditch	Northwestern University, USA	Nodal intersection and geometric control	Let $(M, g)$ be a compact Riemannian manifold and let $H$ be a hypersurface of $M$ . A well-known problem is to restrict the eigenfunctions $\phi_j$ of the Laplacian to $H$ and to get bounds on the $L^2$ norms of the restrictions. My talk is about lower bounds. In general non-trivial bounds don't exist, since an odd function restricts to zero on the fixed point set of an isometric involution. But I will show that if $H$ is asymmetric with respect to geodesics and if almost every geodesic hits $H$ , then there do exist lower bounds for almost all eigenfunctions (for any $\delta > 0$ there exists a subsequence of any ONB of eigenfunctions of density $> \delta$ for which $L^2$ norms of restrictions are $\geq C_{\delta} > 0$ ). The lower bound is applied to give upper bounds on intersections of nodal sets with $H$ in this case.
Junyong	Zhang	Beijing Institute of Technology	Strichartz estimate and Schrodinger operator in a conic singular space	In this talk, we will consider a Schrodinger operator in a conic singular space and discuss the Strichartz estimate for the dispersive equation associated with this operator. The main techniques are the microlocal analysis method and spectral analysis argument. This is a joint work with Jiqiang Zheng (Universite de Nice, France).
Lu	Zhang	Binghamton University, USA	Endpoint estimate for multi-parameter singular Radon Transform	Our work is on an endpoint estimate for the multi-parameter singular Radon Transform. When $p > 1$ , the $L_p$ theory of such operators was first studied by Christ, Nagel, Stein, and Wainger in the single-parameter case, and then by Stein and Street in the multi-parameter setting. We focus on the case when $0 < p \leq 1$ , where we define a Hardy space, and then establish the boundedness of the operators on $H_p$ .
Jiqiang	Zheng	Universite de Nice, France	Low regularity blowup solutions for the mass-critical NLS in higher dimensions	In this talk, we study the $H^s$ -stability of the log-log blowup regime (which has been completely described in a series of recent works by Merle and Raphael) for the focusing mass-critical nonlinear Schrodinger equations $i\partial_t u + \Delta u +  u ^{\frac{4}{d-2}}u = 0$ in $\mathbb{R}^d$ with $d \geq 3$ . We aim to extend the result of Colliander and Raphael for dimension two to the higher dimensions cases $d \geq 3$ , where we use the bootstrap argument and the commutator estimates. This work is jointed with Chenmin Sun.

# Long Time Behavior of Semilinear Tricomi Equations

Daoyin He  
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## Abstract

In this talk, we are concerned with the global Cauchy problem for the semilinear generalized Tricomi equation

$$\partial_t^2 u - t^m \Delta u = |u|^p$$

with initial data  $(u(0, \cdot), \partial_t u(0, \cdot)) = (u_0, u_1)$ , where  $t \geq 0$ ,  $x \in \mathbb{R}^n$  ( $n \geq 2$ ),  $m \in \mathbb{N}$ ,  $p > 1$ , and  $u_i \in C_0^\infty(\mathbb{R}^n)$  ( $i = 0, 1$ ). We show that there exists a critical exponent  $p_{\text{crit}}(m, n) > 1$  such that the solution  $u$ , in general, blows up in finite time when  $1 < p \leq p_{\text{crit}}(m, n)$ . We further show that for the case  $p > p_{\text{crit}}(m, n)$  the solution  $u$  exists globally provided that the initial data is small enough. This result is a generalization of the Strauss's conjecture in degenerate wave equations.

# Scattering of solutions to the 3D defocusing energy sub-critical wave equation

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## Abstract

The defocusing semi-linear wave equation

$$\begin{cases} \partial_t^2 u - \Delta u = -|u|^{p-1}u, & (x, t) \in \mathbb{R}^3 \times \mathbb{R}; \\ u(\cdot, 0) = u_0; \\ u_t(\cdot, 0) = u_1 \end{cases} \quad (CP1)$$

has been extensively studied in the past few decades. This problem is locally well-posed for initial data  $(u_0, u_1)$  in the critical Sobolev space  $\dot{H}^{s_p} \times \dot{H}^{s_p-1}(\mathbb{R}^3)$  with  $s_p \doteq 3/2 - 2/(p-1)$ . Suitable solutions also satisfy an energy conservation law:

$$E(u, u_t) = \int_{\mathbb{R}^3} \left( \frac{1}{2} |\nabla u(\cdot, t)|^2 + \frac{1}{2} |u_t(\cdot, t)|^2 + \frac{1}{p+1} |u(\cdot, t)|^{p+1} \right) dx = \text{Const.}$$

The problem of global existence and scattering is much more difficult to deal with. In the energy critical case  $p = 5$ , M. Grillakis proved that any solution with initial data in the energy space  $\dot{H}^1 \times L^2(\mathbb{R}^3)$  must scatter in both two time directions. It is conjectured that a similar result also holds for other exponents  $p$ : Any solution to (CP1) with initial data  $(u_0, u_1) \in \dot{H}^{s_p} \times \dot{H}^{s_p-1}$  must exist for all time  $t \in \mathbb{R}$  and scatter in both two time directions. In spite of some progress, this conjecture has not been proved yet.

In this talk we show that this conjecture is true if initial data satisfy a couple of additional conditions, i.e. if initial data  $(u_0, u_1)$  are radial so that

$$\|\nabla u_0\|_{L^2(\mathbb{R}^3; d\mu)}, \|u_1\|_{L^2(\mathbb{R}^3; d\mu)} < \infty,$$

where  $d\mu = (|x| + 1)^{1+2\varepsilon} dx$  with  $\varepsilon > 0$ , then the corresponding solution  $u$  must exist for all time  $t \in \mathbb{R}$  and scatter. The key ingredients of the proof include a transformation  $\mathbf{T}$  so that  $v = \mathbf{T}u$  solves another wave equation  $v_{\tau\tau} - \Delta_y v = - \left( \frac{|y|}{\sinh|y|} \right)^{p-1} e^{-(p-3)\tau} |v|^{p-1} v$  with a finite energy, and a Morawetz-type estimate regarding a solution  $v$  as above.

## $L^2$ FOURIER RESTRICTION THEOREMS

JIM WRIGHT

In these lectures we will begin with a survey of the basics of the Fourier restriction problem and its connections and applications with other problems in surrounding areas. For ten years after Stein discovered the Fourier restriction phenomenon, many people attempted to establish the sharp  $L^2$  restriction bound for spheres but then (to quote Stein) *Tomas arrived and removed the shackles from our eyes* and showed us an elegant method to achieve the sharp bound. This evolved into what is now known as the classical Stein-Tomas  $L^2$  restriction argument in which we include the many variants which have arisen since the time of the original arguments. We will review the Stein-Tomas argument and show how it can be formulated in the setting of a general locally compact abelian group. Such formulations have applications with a number-theoretic flavour.

It is well-known that the Stein-Tomas argument tends only to give sharp  $L^2$  Fourier restriction bounds when applied to varieties of codimension 1. In joint work with Jonathan Hickman, we discovered how to formulate a slightly more general version of the Stein-Tomas argument which gives sharp  $L^2$  restriction bounds for a large class of varieties of any dimension. This observation arose as a by-product of work we did concerning certain systems of Diophantine equations. If time permits, we will explore these connections with Diophantine equations more fully.

# Blow-up of a critical Sobolev norm for energy-subcritical and energy-supercritical wave equations

Jianwei YANG

( joint with Thomas Duyckaerts)

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## Abstract

We study the semilinear wave equation

$$(\partial_t^2 - \Delta)u = \iota|u|^{p-1}u, \quad (1)$$

with radial initial data

$$u(0, x) = u_0(x), \partial_t u(0, x) = u_1(x),$$

where  $x \in \mathbf{R}^3$  and  $t \in \mathbf{R}$ . The parameters  $p \in (3, 5) \cup (5, +\infty)$  and  $\iota \in \{\pm 1\}$  are fixed. Let  $m = (p - 1)/2$  and consider the scaling-invariant Sobolev norm  $\mathcal{L}^m$  defined as

$$\|(u_0, u_1)\|_{\mathcal{L}^m} = \left( \int_0^{+\infty} (|r\partial_r u_0(r)|^m + |ru_1(r)|^m) dr \right)^{1/m}. \quad (2)$$

We define an  $\mathcal{L}^m$ -space as the closure of radial, smooth, compactly supported functions for this norm. We develop the local-wellposedness theory in this space by establishing a new Strichartz estimate. Moreover, assume  $(u_0, u_1) \in \mathcal{L}^m$  and  $u$  is a radial solution of the equation (1) with maximal positive time of existence  $T_+$  (in the positive time direction), we obtain the following dichotomy:

- $\lim_{t \rightarrow T_+} \|\vec{u}(t)\|_{\mathcal{L}^m} = +\infty$
- $T_+ = +\infty$  and  $u$  scatters forward in time to a linear solution in  $\mathcal{L}^m$  space.

The same statement holds for the negative time direction. Our strategy is based on new well-posedness theory, profile decomposition adapted to the  $\mathcal{L}^m$ -space, and the *channel of energy* method in the achievement of the *Classification* of radial solutions of the focusing wave equations in the energy critical case: (1) with  $\iota = 1, p = 5$  (the "soliton resolution conjecture" in spherically symmetric case).