

Workshop on Geometry, Analysis and Probability

Oct. 26- Oct 30 at BICMR

	10.26 Th.	10.27 Fri.	10.28 Sat.	10.29 Sun.	10.30 Mon.
9:00-9:50	W. Klingenberg	Yunhui Wu	Chengbo Wang	Longmin Wang	Baoping Liu
10:30-11:20	Zhenlei Zhang	Martin Li	Shiwu Yang	Jingwei Guo	Wenjia Jing
2:00-2:50	Yuan Yuan	Jianquan Ge	Hao Wu	Xin Chen	Discussion
3:30-4:20	S. Pigola	F. Galaz-García	Xiangchan Zhu	Rongchan Zhu	Discussion
4:30-5:30	Weiming Shen		Yanqi Qiu	Dangzheng Liu	

Title and Abstract:

Xin Chen (Shanghai Jiao Tong University)

Title: Estimates and properties of heat kernel for non-local operators with variable order

Let $\alpha(x)$ be a measurable function such that $0 < \alpha_1 \leq \alpha(x) \leq \alpha_2 < 2$ for all $x \in \mathbb{R}^d$, and $\kappa(x, z)$ be a positive measurable function which is bounded from above and below and satisfies that $\kappa(x, z) = \kappa(x, -z)$ for all $x, z \in \mathbb{R}^d$. Under Hölder continuous assumptions on $\alpha(x)$ and $\kappa(x, z)$, we show existence, upper and lower bounds, and regularity properties of heat kernel associated with the following non-local operator with variable order

$$Lf(x) := \int_{\mathbb{R}^d} \left(f(x+z) - f(x) - \nabla f(x) \cdot z \mathbb{I}_{\{|z| \leq 1\}} \right) \frac{\kappa(x, z)}{|z|^{d+\alpha(x)}} dz.$$

This talk is based on a joint work with Zhen-Qing Chen and Jian Wang.

Fernando Galaz-García (Karlsruher Institut für Technologie)

Title: Topology and geometry of three-dimensional Alexandrov spaces

Abstract: Alexandrov spaces are metric generalization of complete Riemannian manifolds with a lower sectional curvature bound. They appear, for example, as (Gromov-Hausdorff) limits of complete Riemannian manifolds with a uniform lower sectional curvature bound. In this talk I will discuss the topology and geometry of these spaces in dimension three, with a view towards open problems and possible research directions.

Jianquan Ge (Beijing Normal University)

Title: DDVV-type inequalities and its applications

Abstract: We will talk about DDVV-type inequalities which are optimal inequalities about sum of square norms of commutators of a sequence of matrices. For some certain types of matrices, these inequalities have good applications to geometry, especially to Simons type inequalities. We will also talk about its generalizations and related problems.

Jingwei Guo (University of Science and Technology of China)

Title: An improved remainder estimate in the Weyl formula for the planar disk

Abstract: Y. Colin de Verdiere proved in 2011 that the remainder term is of order $O(\lambda^{2/3})$ in the two-term Weyl formula for the eigenvalue counting function for the Dirichlet Laplacian associated with the planar disk. In this talk, we will briefly introduce Colin de Verdiere's result and method, and then explain how to improve his bound by using harmonic analysis and prove a new bound $O(\lambda^{2/3-1/495})$.

Wenjia Jing (Yau Mathematical Sciences Center)

TBA

Wilhelm Klingenberg (Durham University)

Title: Lagrangian sections in TS^2

Abstract: Lagrangian surfaces are a tool in the study of the geometry and topology of the ambient symplectic four-manifold. We will review the landmark result of M Gromov from 1985, where he proved that there are no embedded exact Lagrangian tori in \mathbb{R}^4 . In analogy to this we then consider (necessarily exact) Lagrangian spheres in symplectic TS^2 , which arise naturally in classical Differential Geometry. We will prove that the collection of such is locally a Banach manifold near sections with only one complex point. We will conclude by extracting corollaries in Differential Geometry and raise open problems in symplectic geometry of TS^2 .

Martin Man-chun Li (The Chinese University of Hong Kong)

Title: A survey on free boundary minimal surfaces

Abstract: In this talk, we will survey some of the recent developments on the existence and regularity of free boundary minimal surfaces, including in particular some joint work with X. Zhou and N. Kapouleas. Some of these works are partially supported by grants from the Hong Kong Research Grant Council.

Baoping Liu (Beijing International Center for Mathematical Research)

Title: Soliton dynamics in dispersive equations

Abstract: In this talk we focus on the problem of large time behavior for solutions to nonlinear dispersive equations. These equations are typically non-integrable and admit special solutions called solitons. We will discuss the main conjectures and the state of art for various equations.

Dang-Zheng Liu (University of Science and Technology of China)

Title: From random matrices to random operators

Abstract: The local eigenvalue statistics of random band matrices, which interpolate between random Schrodinger operators (Poisson statistics) and random matrices (GOE/GUE statistics), are conjectured to admit an Anderson transition from Poisson statistics to GOE/GUE statistics. We give a brief introduction to such a conjecture.

Stefano Pigola (Universita dell'Insubria)

Title: Potential theory on Riemannian manifolds: some recent results and possible perspectives

Abstract: It is well known that potential theory is a common ground where analysis, geometry and probability meet and interact deeply. In this talk we would like to survey some of these interactions, especially related to parabolicity and to the L^1 -Liouville property for positive superharmonic functions. The emphasis will be placed on the case of manifolds with boundary.

Yanqi Qiu (Chinese Academy of Sciences)

TBA

Weiming Shen (Beijing International Center for Mathematical Research)

Title: On The Negativity of Ricci Curvatures of Complete Conformal Metrics

Abstract: A version of the singular Yamabe problem in bounded domains yields complete conformal metrics with negative constant scalar curvatures. In this talk, I will discuss whether these metrics have negative Ricci curvatures. We will provide a general construction of domains in compact manifolds and demonstrate that the negativity of Ricci curvatures does not hold if the boundary is close to certain sets of low dimension. The expansion of the Green's function and the positive mass theorem play essential roles in certain cases. On the other hand, we prove that these metrics indeed have negative Ricci curvatures in bounded convex domains in the Euclidean space.

Chengbo Wang (Zhejiang University)

Title: Recent progress on the Strauss conjecture and related problems

Abstract: In this talk, we will give an overview of the Strauss conjecture and related problems, as well as the recent development for these problems on various asymptotically flat spacetimes, including nontrapping exterior domains, nontrapping asymptotically Euclidean manifolds, Schwarzschild/Kerr black hole spacetimes. In the process, we discuss some open problems for further investigation.

Longming Wang (Nankai University)

Title: Branching random walks on hyperbolic groups

Abstract: Symmetric branching random walk on a Cayley graph of non-elementary hyperbolic group exhibits a weak survival phase: For growth parameter λ in the interval $[1, R]$ where R is the convergence radius of the underlying random walk, the population survives forever with positive probability, but with probability one, eventually vacates every finite subset of the graph. In this phase, particle trails must converge to a random subset Λ in the geometric boundary of the graph. It is conjectured that the Hausdorff dimension $\Phi(\lambda)$ of the random set Λ has the following behavior as $\lambda \uparrow R$: $\Phi(R) - \Phi(\lambda) \sim C \sqrt{R - \lambda}$ for some positive constant C . In this talk, we will review some progress of this conjecture on Fuchsian groups.

Hao Wu (Yau Mathematical Sciences Center)

Title: Hypergeometric SLE and Convergence of Critical Planar Ising Interfaces

Abstract: Conformal invariance and critical phenomena in two-dimensional statistical physics have been active areas of research in the last few decades. This talk concerns conformally invariant random curves that should describe scaling limits of interfaces in critical lattice models. The scaling limit of the interface in critical planar lattice model with Doburshin boundary conditions (b.c.), if exists, should satisfy conformal invariance (CI) and domain Markov property (DMP). In 1999, O. Schramm introduced SLE process, and this is the only one-parameter family of random curves with CI and DMP. In 2010, D. Chelkak and S. Smirnov proved that the interface of critical Ising model on the square lattice does converge to SLE(3). In this talk, I will discuss the importance of CI and DMP in two-dimensional statistical physics models.

Yunhui Wu (Yau Mathematical Sciences Center)

Title: The Weil-Petersson geometry of the moduli of curves for large genus

Abstract: We study the systole function along Weil-Petersson geodesics. We show that the square root of the systole function is uniform Lipschitz on the Teichmüller space endowed with the Weil-Petersson metric. As an application, we study the growth of the Weil-Petersson inradius of the moduli space of Riemann surfaces of genus g with n punctures as a function of g and n . We show that the Weil-Petersson inradius is comparable to $\sqrt{\ln g}$ with respect to g , and is comparable to 1 with respect to n .

Shiwu Yang (Beijing International Center for Mathematical Research)

Title: Dynamical black holes with prescribed masses in spherical symmetry

Abstract : In this talk, I will review our recent work on a construction of spherically symmetric global solution to the Einstein–scalar field system with large bounded variation norms and large Bondi masses. We show that similar ideas, together with Christodoulou's short pulse method, allow us to prove the following result: Given M_i greater or equal to M_f , there exists a spherically symmetric (black hole) solution to the Einstein scalar field system such that up to an error, the initial Bondi mass is M_i and the final Bondi mass is M_f . Moreover, if one assumes a continuity property of the final Bondi mass (which in principle follows from known techniques in the literature), then for any $M_i > M_f$, this result holds without the error loss. This is the joint work with Jonathan Luk and Sung-jin Oh.

Yuan Yuan (Syracuse University)

Title: Holomorphic isometries between bounded symmetric domains

Abstract: We will first review classical results of Calabi on (local) holomorphic isometries between Kähler manifolds and provide the motivation of holomorphic isometries between bounded symmetric domains. Various recent results on holomorphic isometries between bounded symmetric domains will be discussed, especially the relation to proper holomorphic maps. Some open problems will be given at the end.

Zhenlei Zhang (Capital Normal University)

Title: Kähler-Ricci flow on smooth minimal elliptic surfaces

Abstract: It has been known since Song-Tian 2006 that the Kähler-Ricci flow on a smooth minimal elliptic surface converges in the current sense to a canonically defined generalized Kähler-Einstein metric on the canonical model. They also conjectured that the convergence also take place geometrically in the Gromov-Hausdorff topology. It is the simplest collapsing structure in the Analytic Minimal Model Program on minimal models. In this talk I will present the confirmation to this conjecture. It is a joint work with Professor Tian.

Rongchan Zhu (Beijing Institute of Technology)

Title: Stochastic Heat Equations Taking Values in a Riemannian Manifold

Abstract: In this talk, I talk about the existence of martingale solutions to the stochastic heat equation in a Riemannian manifold by using suitable Dirichlet form on the Riemannian path and loop space. Finally, by using Anderson-Driver's Approximation, we give a form of the equation associated with the process given by Dirichlet form.

Xiangchan Zhu (Beijing Jiaotong University)

Title: Recent results on stochastic quantization for Φ^4_d model

Abstract: In this talk, I give a survey on recent development for stochastic quantization of Φ^4_d model. We mainly talk about the strong approach, i.e. regularity structure theory and paracontrolled distribution method, and the weak approach, i.e. Dirichlet form theory. Moreover, we will explain the relation between the above two methods.