

# STABILITY, BOUNDEDNESS AND FANO VARIETIES

## 1. LECTURE SERIES

**Caucher Birkar** (University of Cambridge)

Title: Boundedness of singularities and Fano varieties

Abstract: The first lecture will be an introduction to recent results on local and global boundedness properties of singularities of linear systems and of Fano varieties. Both singularities and Fano varieties are central topics in birational algebraic geometry, especially in the minimal model program. In the second and third lectures I will try to describe some of the ideas in the proof of the main results with particular attention given to effective birationality, complements, and BAB.

**Mattias Jonsson** (University of Michigan)

Title: A non-Archimedean approach to K-stability and the existence of Kähler-Einstein metrics

Abstract: The recently proved Yau-Tian-Donaldson (YTD) conjecture says that a Fano manifold admits a Kähler-Einstein metric if and only if it is K-(poly)stable. The latter is a condition on certain kinds of degenerations of  $X$ , called test configurations, which, unfortunately, is rather difficult to verify in practice.

My goal in these lectures is two-fold. First, I will survey recent work by several people that gives a characterization/interpretation of K-stability in terms of valuations and non-Archimedean geometry. Second, I will outline joint work with Berman and Boucksom where we use these considerations, together with some techniques from potential theory, to give a variational proof of the YTD conjecture.

## 2. ONE HOUR TALKS

**Hamid Ahmadinezhad** (Loughborough University)

Title: Birational geometry of del Pezzo fibrations and stability conditions

Abstract: We are interested in characterising birationally rigid del Pezzo fibrations. In this talk, I explain some conjectural statements for birational rigidity of low degree fibrations, and I provide a counterexample. Then I spell out a stability condition that can save the conjecture. I will report on progress on this project, which is joint work with Igor Krylov and Maksym Fedorchuk.

**Florin Ambro** (Romanian Academy)

Title: Curves with ordinary singularities

Abstract: I will discuss the classification of projective curves with ordinary singularities (i.e. seminormal), in a way parallel to the classification of projective curves with no singularities.

**Meng Chen** (Fudan University)

Title: On anti-canonical geometry of weak  $\mathbb{Q}$ -Fano threefolds (Part II)

Abstract: By a canonical (resp. terminal) weak  $\mathbb{Q}$ -Fano 3-fold we mean a projective threefold with at worst canonical (resp. terminal) singularities on which the anti-canonical divisor is  $\mathbb{Q}$ -Cartier, nef and big. For a canonical weak  $\mathbb{Q}$ -Fano 3-fold  $V$ , we show that there exists a terminal weak  $\mathbb{Q}$ -Fano 3-fold  $X$ , being birational to  $V$ , such that the  $m$ -th anti-canonical map, defined by  $|-mK_X|$ , is birational for all  $m \geq 52$ . This is a joint work in progress with Chen Jiang.

**Sung Rak Choi** (Yonsei university)

Title: Okounkov bodies associated to pseudoeffective divisors.

Abstract: An Okounkov body is a convex set in the Euclidean space associated to a divisor and it is predicted that those bodies capture many of the positivity of divisors. It is still mysterious and not fully known how the positivity of divisors are encoded in the Okounkov bodies. In this talk, I will explain how to recover some of the positivity of pseudoeffective divisors from Okounkov bodies. I will also try to present some birational behaviors of the Okounkov bodies.

**Osamu Fujino** (Osaka University)

Title: Bertini type theorem for multiplier ideal sheaves

Abstract: Let  $\varphi$  be a quasi-plurisubharmonic function on a compact complex manifold  $X$ . It is well known that the multiplier ideal sheaf  $\mathcal{I}(\varphi)$  associated to  $\varphi$ , which was introduced by Nadel, is an important object. In this talk, I would like to explain a Bertini-type theorem for  $\mathcal{I}(\varphi)$ . Let  $H$  be a smooth general member of a free linear system  $\Lambda$  on  $X$ . Then the natural inclusion  $\mathcal{I}(\varphi|_H) \subset \mathcal{I}(\varphi)|_H$  always hold by the Ohsawa–Takegoshi  $L^2$  extension theorem. Unfortunately, the equality  $\mathcal{I}(\varphi|_H) = \mathcal{I}(\varphi)|_H$  does not always hold. However, I claim that there are many  $H$  in  $\Lambda$  satisfying  $\mathcal{I}(\varphi|_H) = \mathcal{I}(\varphi)|_H$ .

**Kento Fujita** (RIMS)

Title: Openness results for uniform K-stability

Abstract: It is expected that several "openness" results holds for uniform K-stability of polarized varieties. In this talk, I will discuss certain partial results for this problem.

**Chen Jiang** (IPMU, Tokyo)

Title: Boundedness of K-semistable  $\mathbb{Q}$ -Fano varieties with degrees bounded from below.

Abstract: Applying recent development in birational geometry, we show that  $\mathbb{Q}$ -Fano varieties of fixed dimension with anti-canonical degrees and alpha-invariants bounded from below form a bounded family. As a corollary, K-semistable  $\mathbb{Q}$ -Fano varieties of fixed dimension with anti-canonical degrees bounded from below form a bounded family.

**Vladimir Lazić** (Saarland University)

Title: Two conjectures in birational geometry

Abstract: I will discuss recent progress on two conjectures in birational geometry: the nonvanishing conjecture and a conjecture of Mumford. This is joint work with Thomas Peternell.

**Diletta Martinelli** (University of Edinburgh)

Title: On the number and boundedness of log minimal models of a variety of general type

Abstract: I will talk about a recent joint project with Stefan Schreieder and Luca Tasin where we prove that minimal models of general type of given dimension and bounded volume form a bounded family (if time permits I will also talk about the klt case). Moreover, we prove that the number of minimal models can be bounded by a constant depending only on the canonical volume. In the end, I will show that in some cases for threefolds it is possible to give some effective bounds for this number.

**Jesus Martinez Garcia** (University of Bath)

Title: Stability and cscK metrics on polarised del Pezzo surfaces

Abstract: The Yau-Tian-Donaldson conjecture relates the existence of constant scalar curvature Kähler metrics on polarised manifolds to the algebro-geometric notion of K-stability. In this introductory talk we will describe the conjecture and the difficulty to test either side of the equivalence. Then I will focus on the case of del Pezzo surfaces for which I will provide several obstructions using birational geometry. This is joint work with Ivan Cheltsov.

**Takuzo Okada** (Saga University)

Title: On stable rationality problem of Fano varieties

Abstract: I will talk about stable rationality problem of algebraic varieties with a focus on Fano varieties. After reviewing recent results, I will explain the failure of stable rationality of some Fano varieties such as smooth weighted hypersurfaces and index one Fano hypersurfaces containing a linear subspace multiply both in arbitrary dimension.

**Jason Starr** (Stony Brook University)

Title: Symplectic Invariance of Rational Surfaces on 2-Fano Manifolds

Abstract. The 2-Fano manifolds are those Fano manifolds whose second graded piece of the Chern character is positive, e.g., smooth, degree  $d$  hypersurfaces in projective  $n$ -space with  $d^2 \leq n$ . These Fano manifolds are swept out by rational surfaces. The enumerative invariants of rational surfaces on a Fano manifold are not symplectically invariant. Nonetheless, if a certain combination of

gravitational descendants is positive, then every complex projective manifold that is symplectically deformation equivalent to the Fano manifold is also swept out by rational surfaces.

**Luca Tasin** (University of Bonn)

Title: Kähler structures on spin 6-manifolds Abstract. A smooth 6-manifold  $M$  is said to be Spin if its second Stiefel-Whitney class is zero. If  $X$  is a complex structure on  $M$ , the Spin condition is equivalent to say that  $K_X$  is divisible by 2 in  $H^2(X, \mathbb{Z})$ . Since this depends only on the homotopy type of  $M$ , it is a natural question to study the possible Kähler structures on  $M$ . In this talk I will present several finiteness and boundedness results on such structures, which are based on the study of their birational properties.

**Zhiyu Tian** (CNRS, Grenoble)

Title: Crepant resolution conjecture, Chow motive, and hyperkahler manifolds

Abstract: The crepant resolution conjecture relates the GW theory of a stack with that of a crepant resolution of its coarse moduli. I will make some remarks about lifting this to Chow motives in the global quotient stack case and discuss some applications to the study of the Chow ring of some hyperkahler manifolds. This is based on joint work with Lie Fu and with Lie Fu-Charles Vial.

**Xiaowei Wang** (Rutgers University, Newark)

Title: Moduli space of Fano Kähler-Einstein varieties

Abstract: In this talk, we will discuss our construction of proper schemes as moduli spaces parametrizing smoothable K-stable Fano varieties. The solution relies on the recent solution of the Yau-Tian-Donaldson conjecture by Chen-Donaldson-Sun and Tian. In particular, we prove the uniqueness of the degeneration of Fano Kähler-Einstein manifolds and more algebraic properties that are needed to construct a good algebraic moduli space. (This is a joint work with Chi Li and Chenyang Xu)

**Qizheng Yin** (BICMR)

Title: Derived categories of K3 surfaces, OGrady's filtration, and cubic fourfolds

We present recent joint work with Junliang Shen and Xiaolei Zhao relating

1) the derived category of a K3 surface 2) zero-cycles on the K3 surface 3) zero-cycles on the moduli spaces of stable objects in the derived category.

We also explain how to extend the relations above to the K3 category of a cubic fourfold. The latter is joint work in progress with Junliang Shen.

**Ziquan Zhuang** (Princeton University)

Title: Fano varieties with large Seshadri constants

Abstract: Given a Fano variety, a natural way to measure its local positivity is through the Seshadri constants of its anticanonical divisor. In this talk, we will explore how having large Seshadri constants restricts the geometry of the Fano variety. For example, we'll show that an  $n$ -dimensional Fano variety is isomorphic to  $\mathbb{P}^n$  if the Seshadri constant of its anti-canonical divisor at some smooth point is greater than  $n$  (in the smooth case, this is a result of Bauer and Szemberg) and that those with Seshadri constants  $n$  always have klt singularities. We will also prove that the set of Fano varieties with Seshadri constants greater than  $n - 1 + c$  (resp.  $n - 1$ ) for some  $c > 0$  is weakly (resp. birationally) bounded. Part of this work is joint with Yuchen Liu.